OPTIMIZATION MODELS APPLIED TO EQUIPMENT REPLACEMENT POLICY

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The Equipment Replacement Policy (ERP) represents a classical investment analysis decision with significant impacts on the performance of companies. This study proposes a hybrid mathematical model in order to support long term planning through the use of Engineering Economy, Vehicle Routing Problem (VRP) and Dynamic Programming (DP) as alternative procedures for evaluating the best replacement policy. The proposed model was tested using two non-identical fixed assets. The results were compared to other usual techniques such as Shortest Path (SP), Dynamic Programming and Equivalent Annual Cost and suggest that is a useful alternative methodology.

Palavras-chaves: Engineering Economy, Vehicle Routing Problem and Dynamic Programming
1. INTRODUCTION
The replacement of fixed assets such as machinery and equipment is due to the following reasons:

a) High operational and maintenance costs because of physical deterioration;
b) Technological obsolescence that limits more competitive and better quality products;
c) Leasing of similar equipment at competitive rates;
d) The inability to meet present demand.

Both late and early replacement of machinery and equipment can lead companies to incur financial losses either on their capital return or high operational costs, respectively (VALVERDE & RESENDE, 1997). Therefore, from an investment or productivity perspective, the replacement of fixed assets represents a typical and relevant financial decision that affects the life of the company for several years. In many cases, the decision must be taken before the end of the useful life of the existing equipment.

The main problem is to keep existing equipment or to replace it with a more competitive alternative. In general, the subject can be divided into two parts (FLEISCHER, 1987):

a) A write-off with identical equipment, when the new equipment has an initial cost, operational costs, working life and market value similar to the existing equipment;
b) A write-off with non-identical equipment, when the new equipment has different characteristics from the existing equipment (e.g.: the initial cost, operational costs and working life).

Other variables affect the analysis such as: the horizon planning, the number of possible replacements during HP and the nature of the cash flow. Table 1 summarizes the possible combinations according to the defined variables and gives the suggested solution techniques, such as Integer Linear Programming (ILP), Dynamic Programming (DP) and Simulation.

In thesis, according to economic assessments, many replacements could be done during the horizon planning. However, the nature of the analyzed fixed assets (size, complexity), available budget and company’s replacement policy defines or limits possible replacements. Fixed assets, such as a steel rolling mill, hydraulic press, hydroelectric turbines and other such equipment make short term replacement impossible. On the other hand, other fixed assets allow for more frequent replacement (e.g.: vehicles).

Both of the above categories need data and estimations of the planning horizon, operational costs, depreciation, cost of capital or minimum interest rate, working life of the analyzed equipment, market values etc. Thus, the replacement problem is a complex decision-making activity. From the perspective of the optimization methods, generically, there are many studies on this subject that apply Dynamic Programming (NAIR & HOPP, 1992; LADANY, 1997; VALVERDE & RESENDE, 1997; DOGRAMACI & FRAIMAN, 2004; MARQUES et al, 2005), Integer Linear Programming (TAHA, 2008) or both (AHUJA et al, 1993; AKBALIK et al, 2008; TAHA, 2008).
Table 1 – Suggested available techniques applied to the optimal replacement policy of an existing fixed asset against the best alternative during the horizon planning (HP)

Generally, the specific literature gives examples of a write-off with identical equipment, but this is an unreal category. The write-off with non-identical equipment is more important and realistic. The objective of this work is to propose a hybrid mathematical economic assessment model of existing equipment against non-identical equipment, over a long-term horizon, with many possible replacements and a deterministic cash flow based on the concepts of Engineering Economy, Vehicle Routing Problem (VRP) and Dynamic Programming. As many engineering decisions are based on costs, the selected objective function is cost minimization. The proposed method was tested with data from the problem presented by Hirschfeld (2009) and the results were compared to the Shortest Path model, Dynamic Programming and Equivalent Annual Cost technique. The simulations were developed in an Excel electronic sheet, using the Solver optimization software from Microsoft Excel®.

The article is divided into four parts: (i) describes the concepts of Engineering Economy, Shortest Path Problem, Vehicle Routing Problem, Equipment Replacement Policy, Dynamic Programming; (ii) the proposed model is presented; (iii) the tests and results are shown and (iv) the conclusions are presented.

2. FUNDAMENTALS
2.1 Engineering Economy applied to Equipment Replacement Policy

The economic analysis of fixed asset replacement is a project composed of many parts such as: (i) age of the existing equipment; (ii) its remaining working life; (iii) accounting costs (e.g.: maintenance, materials, depreciation); (iv) economic costs (e.g.: opportunity cost); (v) horizon planning; (vi) taxes; (vii) initial cost of the new equipment (initial investment); (viii) market value of existing equipment and (ix) discount rate of the analyzed cash flow.
At the beginning of the process (today), generally, the initial cost of the existing equipment, its age and its remaining working life are known and the working life can be obtained by carrying out technical studies or from legal tables. Horizon planning or service life is the limit of the economic life of the existing equipment. Thus, the horizon planning works as a window that contains the analyzed cash flow (HIRSCHFELD, 2009).

The cost approach during horizon planning represents the conflict between accounting and economic costs. Accounting costs account current expenses (human resources, materials, maintenance) plus depreciation (decrease in value due to physical deterioration or obsolescence) controlled by current legislation. Economic costs consider opportunity cost or the unprofitable way of using the resources (PINDK & RUBINFELD, 2005). They do not necessarily have an account record. In the replacement policy, an example of an economic cost would be the opportunity of not selling the existing equipment. Both costs must be considered from the moment of replacement and do not consider past costs. Due to low liquidity, which is characteristic of fixed assets, which limits the short-term selling possibilities, the account criteria was adopted to form the cash flow including operational costs (human resources, materials, maintenance), depreciation and its benefits.

During horizon planning both pieces of equipment (existing and new) can be evaluated and their respective market value assessed according to secondary markets (e.g.: vehicles), specific studies and by accounting depreciation methods. The market value must include the existing cash flow after horizon planning if its working life is greater than the service life. During the analysis, the market value of the existing equipment is considered as a reduction in the initial cost of the new equipment.

The tax rate affects the selling result, the operational costs (reducing the value) and the depreciation (HIRSCHFELD, 2009; ROSS et al, 2008). Depreciation is an important part of the fixed asset cash flows because those pieces of equipment are more influenced by physical deterioration than technological obsolescence. It is an economic expense but has tangible legal benefits by reducing the tax value. Moreover, in accordance with the operational basis (e.g.: 3 shifts per day), it is possible to use greater quotas, thereby increasing its benefits.

The interest rate is another relevant aspect. The main idea is related to the concepts of cost of capital, in other words, a project creates value if its profit is greater than the cost of capital used to support its activities. In order to carry out their projects companies use a great variety of funds, their own money or the money of others, all with different costs. Thus, the minimum attractive rate (MAR) should be the average of these costs or a weighted average cost of capital (BRIGHAM & EHRHARDT, 2007).

Figure 1 shows a cash flow of an existing piece of four-year old equipment, initial cost $C_a_0$, operational costs $C_i$, 10 years working life and market value $V_R_0$. The new equipment has an initial cost $I_0$, 10 years working life, operational costs $C_i$ and market value $V_R_6$ assessed during the horizon planning including the cash flow after its service life. The cash flow is discounted by the cost of capital.
2.2 Shortest Path Problems

Shortest path problems are relevant for several reasons: (i) they arise frequently in practice since in a wide variety of application settings we wish to send some material between two specified points in a network as quickly, as cheaply or as reliably as possible; (ii) they are easy to solve efficiently; (iii) they provide both a benchmark and a point of departure for studying more complex network models and (iv) they arise frequently as subproblems when solving many combinatorial and network optimization problems. As examples of shortest path models are: urban housing, project management, inventory planning, DNA sequencing, product planning, telephone operator scheduling and vehicle fleet planning (AHUJA et al, 1993).

Considering a directed network $G = (N=nodes, A=arcs)$ with an arc length $c_{ij}$ associated with each arc $(i,j) \in A$. The node $s$ is called source. We define the length of a directed path as the sum of the lengths (or costs) of arcs in the path. The problem is to determine the shortest length directed path from node $s$ to node $i \in N$. Several assumptions are imposed such as: (i) all arc lengths are integers; (ii) the network contains a directed path from node $s$ (source) to every other node in the network; (iii) the network does not contain a directed cycle of negative length and (iv) the network is directed i.e., if $x_{ij}$ exists $x_{ji}$ does not exist. A general linear programming formulation is presented below (AHUJA et al, 1993).
min \( Z = \sum_{(i,j) \in A} c_{ij}x_{ij} \)

subject to:
\[
\sum_{(j(i,j) \in A)} x_{ij} - \sum_{(j(i,j) \in A)} x_{ji} = \begin{cases} n-1 & \text{for } i = s \\ -1 & \text{for all } i \in N - \{s\} \end{cases}
\]

\( x_{ij} \geq 0 \quad \forall (i, j) \in A \)

\( x_{ij} \geq 0 \) and integer

### 2.3 Vehicle Routing and Equipment Replacement Problem

A routing system is defined as a group of organized means in order to meet demands located on the arcs or nodes of a transportation network (GOLDBARG & LUNA, 2005). VRP generally treats tours over demand or offering points. Those points can represent cities, deposits, posts of work etc.

The VRP can be represented as the following graph-theoretic problem. Let \( G = (V, A) \) be a complete graph where \( V = \{0, 1, \ldots, n\} \) is the vertex set and \( A \) is the arc set. Vertices \( j = 1, \ldots, n \) correspond to the customers, each with a known non-negative demand, \( d_j \), whereas vertex 0 corresponds to the depot. A non-negative cost, \( c_{ij} \), is associated with each arc \((i, j) \in A\) and represents the cost of traveling from vertex \( i \) to vertex \( j \). If the cost values satisfy \( c_{ij} = c_{ji} \) for all \( i, j \in V \), then the problem is said to be a symmetric VRP; otherwise, it is called an asymmetric VRP (EKSIOGLU et al, 2009).

Fisher & Jaikumar (1981) developed one of the most popular VRP model as shown below.

\[
\text{min } Z = \sum_{i,j} c_{ij} \sum_{k=1} y_{ijk}
\]

subject to:

\[
\sum_{k} y_{ik} = 1 \quad i = 2,\ldots,n \quad (1)
\]

\[
\sum_{k} y_{ik} = m \quad i = 1 \quad (2)
\]

\[
q_i y_{ik} \leq Q_k \quad k = 1,\ldots,m \quad (3)
\]

\[
\sum_{j} x_{ijk} = \sum_{j \in S} y_{ijk} = y_{ik} \quad i = 1,\ldots,n \quad k = 1,\ldots,m \quad (4)
\]

\[
\sum_{i, j \in S} x_{ijk} \leq |S| - 1 \quad \forall S \subseteq \{2,\ldots,n\}, k = 1,\ldots,m \quad (5)
\]

\( x_{ijk} \in \{0,1\} \quad i = 1,\ldots,n \quad k = 1,\ldots,m \)

\( y_{ik} \in \{0,1\} \quad i, j = 1,\ldots,n \quad k = 1,\ldots,m \)
Where:

- \( x_{ijk} \) = variable that assumes 1 when the vehicle \( k \) visits customer \( j \) after visited customer \( i \) or 0 on the contrary
- \( y_{ik} \) = variable that assumes 1 if customer \( i \) is visited by vehicle \( k \) or 0 on the contrary
- \( q_i \) = demand of customer \( i \)
- \( Q_k \) = capacity of vehicle \( k \)
- \( c_{ij} \) = cost of the arc \( ij \)
- \( S \) = legal circuits in accordance with the number of nodes

ERP can be considered as a type of VRP given the following reasons:

a) ERP can be developed in a net representation with extreme nodes representing the beginning and the end of the process;

b) The intermediate nodes represent the states of retention and replacement at a current stage from states from previous stages;

c) Vehicle of VRP model can be seen as the chosen type of equipment of ERP that runs a route of sequential nodes containing arcs \( ij \);

d) ERP and VRP are Hamilton’s cycles. The pieces of equipment (vehicles) realize their tours through the arcs (length, states, decisions) visiting each node (cities, deposits, stages) in the network exactly once;

e) At each stage, the nodes represent the demand to be provided with the capacity of each equipment (vehicle);

f) The ERP network presents no negative length (arc);

g) At each stage, the decision of retention or replacement the equipment is irreversible. Therefore the ERP network is directed i.e. as \( x_{ij} \) is a binary variable then \( x_{ij} + x_{ji} = 1 \);

h) The arcs represent the transition values (costs) between different stages and the nodes represent the restrictions of the stage (demand, capacity);

i) The purpose is to find the route which maximizes (profit) or minimizes (cost) the analyzed objective function;

j) In accordance with VRP model, the optimal ERP solution is a continuous route.

2.4 Dynamic Programming

Dynamic Programming (DP) is a method applied to multivariable problems that can be divided into a sequence of stages, each one representing a small part of the problem with only one variable. The options for concluding each stage are called decisions. A policy is a sequence of decisions, with one decision for each stage of the process. The situation of the process at a certain stage is called its state at this stage. Each decision affects the transition from the present stage to the related state of the following stage. Basically, a DP model is a recurring equation that connects different stages of the problem in order to assure that the optimal solution for each stage is the optimal solution for the whole problem (HILLIER and LIEBERMAN, 2006; TAHA, 2008). The DP terminology used is presented below.

**Horizon Planning:** remaining working life of the existing equipment which corresponds to the number of stages.

**Stages:** the years of each replacement decision.
State: the situation of the system (keep or replace) at a certain stage.

Transition: is defined by the recurring equation representing the change of variable values from a state at a certain stage to the state of the following stage. According to the replacement problem, transition values represent the cost or the revenue cash flow involved with keeping or replacing the existing equipment. Under the cost perspective, the negative discounted cash flow is composed of: (i) operational costs \(c\) and (ii) the initial cost of the new equipment \(I\). Positive cash flow is composed of the market value of the analyzed equipment \(VR\).

Decision: keep or replace the existing equipment. The basic idea is to replace the equipment when the cost of operating it becomes high enough in net present value terms to justify a replacement.

Policy: a set of long term decisions throughout the horizon planning

- \(N\) = horizon planning or number of stages
- \(t\) = age of the equipment at the beginning of the stage
- \(I_0\) = initial cost of the best market alternative
- \(C_j\) = annual operational cost at the beginning of the stage
- \(D_j\) = depreciation quota
- \(VC_j\) = the equipment’s accounting value at the beginning of the stage
- \(VR_j\) = the equipment’s market value
- \(r\) = cost of capital (% per year)
- \(IR\) = taxes (%)

Assuming that the existing equipment is \(t\) years old at the beginning of stage (year) \(j\) and the cash flow is represented by annual values, the recurring equation is:

\[
f_j(t_j) = \min \left\{ \left[ -C(t_j)(1-IR) + D_j(1+IR) \right](1+r)^{-1} + \sum_{i=1}^{n} \sum_{k=1}^{A} C_{jik} x_{jik} \right\} \quad \text{Keep}
\]

\[
f_{j+1}(t_{j+1}) = \min \left\{ \left[ -(C(0) + VR_j - VC_j - I_0)(1-IR) + D_j(1+IR) \right](1+r)^{j} + \left[ f_{j}(t_j) \right] \right\} \quad \text{Replace}
\]

\(f_{0}(t_0) = 0\)

3. THE PROPOSED MODEL

The characteristics of the replacement problem simplify the optimization process because they reduce the number of identified arcs. Among many available vehicle routing problem solutions, the model developed by Fisher & Jaikumar (1981) was chosen given the following reasons:

- a) It is a classical and generic model used in many other approaches (LAPORTE et al, 2000);
- b) Is an Integer Linear Programming with exact mathematical methods such as Branch and Bound that is presented in many software;
- c) By allowing integer-value restrictions on decision variables, it is possible to include more sophisticated logical constraints such as the maximum number of replacements during the horizon planning and demand restriction;
- d) The model was tested under conflict with many vehicles in the same net. This characteristic should be useful for future expansion of the proposed model.

The mathematical representation of the possible arcs of the model according to the features presented is:

\[
\max Z = \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{k=1}^{A} c_{jik} x_{jik}
\]
(minimum cost)

(route continuity restriction)

(demand restriction)

(maximum number of replacements)

Where:

\[ x_{ijk} = \text{arc from } i \text{ to } k \text{ utilized by equipment } j \]

\[ y_j = \text{shows that node } i \text{ is part of the solution} \]

\[ J = \text{number of pieces of equipment} \]

\[ W = \text{number of nodes} \]

\[ A = \text{arcs of the stage} \]

\[ M = \text{set of replacement nodes} \]

\[ S = \text{number of stages} \]

\[ n = \text{number of arcs} \]

\[ Q_j = \text{capacity of equipment } j \]

\[ d_s = \text{demand of stage } s \]

\[ R = \text{maximum number of replacements during the horizon planning} \]

\[ t = \text{age of the equipment at the beginning of the stage} \]

\[ I_0 = \text{initial cost of the best market alternative} \]

\[ C_j = \text{the annual operational cost at the beginning of the stage} \]

\[ D = \text{depreciation quota} \]

\[ VC = \text{the equipment’s accounting value at the beginning of the stage} \]

\[ VR = \text{the equipment’s market value} \]

\[ r = \text{cost of capital (\% per year)} \]

\[ IR = \text{taxes (\%)} \]

As explained, the proposed model is based on VRP features. The route intends to minimize the present value of the total cost (objective function) during the horizon plan. The route continuity restriction guarantees, at each stage, that the chosen equipment (vehicle) visits the node exactly once. The sum of chosen replacement nodes \( y_w \) is equal or smaller than the maximum number of replacements. At each stage, the capacity of the chosen equipment is equal or greater than the demand. In accordance with the characteristics of ERP, the VRP restrictions (1), (2) and (5) are not necessary.

4. RESULTS

The problem presented by Hirschfeld (2009) was adapted to evaluate the proposed model.
Equipment $K$ bought four years ago, presented the following features: (i) initial cost US$ 1,000,000; (ii) 10 years working life; (iii) annual maintenance costs US$ 100,000; (iv) market value after 10 years US$ 150,000. The company intends to replace it with equipment $L$ which has the following characteristics: (i) initial cost US$ 1,500,000; (ii) 10 years working life; (iii) annual maintenance costs US$ R$ 80,000; (iv) market value after 10 years US$ 250,000. The seller of $L$ offers US$ 400,000 for $K$. The company considers 20% per year as its capital cost and 35% as taxes. The demand at each stage is 3,000 tons and the capacity of equipment $L$ and equipment $K$ are, respectively, 3,500 and 3,000 tons. One replacement is predicted during the horizon planning. Table 2 shows the necessary data for applying the optimization techniques from the beginning of the analysis until the end of the horizon planning.

Table 3 and Figure 2 below were formed according to the models presented in section 2. They summarize and compare Shortest Path model, Dynamic Programming, Equivalent Annual Cost (EAC) and the proposed model. The results show that retention is the best policy. This was the same result found by using the Equivalent Annual Cost technique (HIRSCHFELD, 2009). The principles of EAC were presented in section 2.1, moreover, EAC converts each cash flow event into a uniform series of payments (HIRSCHFELD, 2009).

<table>
<thead>
<tr>
<th>Age</th>
<th>Existing Equipment</th>
<th>New Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Operation Cost $C(t)$</td>
<td>Account Value $VC$</td>
</tr>
<tr>
<td>1</td>
<td>100,000</td>
<td>600,00</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>100,000</td>
<td>400,00</td>
</tr>
<tr>
<td>5</td>
<td>100,000</td>
<td>320,00</td>
</tr>
<tr>
<td>6</td>
<td>100,000</td>
<td>280,00</td>
</tr>
<tr>
<td>7</td>
<td>100,000</td>
<td>240,00</td>
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<tr>
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<td>100,000</td>
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</tr>
<tr>
<td>9</td>
<td>100,000</td>
<td>160,00</td>
</tr>
<tr>
<td>10</td>
<td>100,000</td>
<td>150,00</td>
</tr>
</tbody>
</table>

Source: prepared by the author

1. estimated value from available data
2 linear depreciation
3 including cash flow after horizon planning

Table 2 – Operational costs, market value and depreciation

<table>
<thead>
<tr>
<th></th>
<th>Proposed VRP Model</th>
<th>Shortest Path Model</th>
<th>Dynamic Programming</th>
<th>Equivalent Annual Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>67,112</td>
<td>67,112</td>
<td>67,112</td>
<td>67,112</td>
</tr>
<tr>
<td>Advantages</td>
<td>Flexible Optimization technique</td>
<td>Flexible Optimization technique</td>
<td>Traditional method Optimization technique</td>
<td>Traditional method Practical</td>
</tr>
<tr>
<td>Disadvantages</td>
<td>Net size</td>
<td>Net size</td>
<td>Net size</td>
<td>It does not assess intermediate replacement possibilities</td>
</tr>
<tr>
<td></td>
<td>It does not consider capacity restriction</td>
<td>It does not consider capacity restriction</td>
<td>It does not consider maximum number of replacements</td>
<td></td>
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<td></td>
<td>It does not consider maximum number of replacements</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Hirschfeld (2009)
2 Present value

Table 3 – Results and comparison of analyzed methods
5. CONCLUSIONS
ERP has been researching for decades (BELLMAN, 1955). Mathematical models and solutions for similar problems are long known (dynamic programming). The generalized VRP provides a useful modeling framework for a wide variety of applications (BALDACCI et al, 2009). According to reasons presented, this study proposes a hybrid vehicle routing model as an alternative approach to equipment replacement policy problem.
This study reviewed the replacement of fixed assets from the perspective of optimization methods and its connections with Engineering Economy and other optimization problems. This assessment presented an alternative approach to non-identical assets and relevant financial aspects such as depreciation, cost of capital and taxes, generally, treated in a simplified way in order to form realistic cash flows. An alternative model based on VRP and Integer Linear Programming was developed and compared to other techniques. General and specific advantages were identified and are listed below:

a) No previous study has ever proposed the use of VRP as an alternative solution of ERP. It is a new perspective of a classical problem that presents a coherent and an appropriate approach that eases the understanding of ERP;

b) It is a practical model that can be used with popular software. In this work, the model was developed in an Excel electronic worksheet using the Solver optimization software from Microsoft Excel®;

c) The proposed model is flexible and can be adjusted to work, simultaneously, with budget restrictions, a maximum number of replacements, the company’s replacement policy limitations and demand restrictions.

The size of the net presents an operational limitation. However, current information technology resources allow the solution of many real problems. Regarding the natural evolution of these resources it is easy to predict an increase in possible applications. This work intends to motivate new studies and improvements in the model, such as the optimal policy of multiple equipment alternatives compared to the existing equipment.

REFERENCES


